Revealing and Incorporating Implicit Communities to Improve Recommender Systems

EUIJIN CHOO, North Carolina State University
TING YU, North Carolina State University; Qatar Computing Research Institute
MIN CHI, North Carolina State University
YAN LINDSAY SUN, University of Rhode Island

Social connections often have a significant influence on personal decision making. Researchers have proposed novel recommender systems that take advantage of social relationship information to improve recommendations. These systems, while promising, are often hindered in practice. Existing social networks such as Facebook are not designed for recommendations and thus contain many irrelevant relationships. Many recommendation platforms such as Amazon often do not permit users to establish explicit social relationships. And direct integration of social and commercial systems raises privacy concerns.

In this paper we address these issues by focusing on the extraction of implicit and relevant relationships among users based upon the patterns of their existing interactions. Our work is grounded in the context of item recommendations on Amazon. We investigate whether users' reply patterns can be used to identify these meaningful relationships and show that different degrees of relationships do exist. We develop global measures of relationship strength and observe that users tend to form strong connections when they are evaluating subjective items such as books and movies. We then design a probabilistic mechanism to distinguish meaningful connections from connections formed by chance and extract implicit communities. We finally show that these communities can be used for hybrid recommender systems that improve recommendations over existing collaborative filtering approaches.

Categories and Subject Descriptors: C.2.4 [**Computer Communication Networks**]: Distributed Systems-Distributed applications; K.4.4 [**Computer and Society**]: Electronic Commerce

Additional Key Words and Phrases: Recommender system, implicit communities, recommendation, social network

1. INTRODUCTION

Online users are often overwhelmed by a deluge of available information and alternative choices. Recommender systems are designed to enable users to channel this flow and to make efficient choices, identifying personally-relevant items in an otherwise unending storm. These systems gather individual users' preferences and then use that information to recommend appropriate *items* to them. These items can be any object that is consumed or evaluated by users such as products for sale (e.g. Amazon or Ebay listings), or sources of information (e.g. news articles on Reddit or Boing Boing).

A number of recommender systems have been proposed in the literature [Funakoshi and Ohguro 2000; Linden et al. 2003; Pazzani and Billsus 2007; Resnick and Varian 1997; Sarwar et al. 2001; Schafer et al. 2007]. In recent years, researchers have begun to pay attention to the role that social connections can play in making recommendations [Arazy et al. 2009; Bonhard and Sasse 2006; DuBois et al. 2009; He and Chu 2010; Jiang et al. 2012; Ma et al. 2011]. In general, people who have difficulty making a decision often ask friends or colleagues for advice which, in turn, plays an important role in their final decision. Social recommender systems have been shown to be more effective for some applications than collaborative filtering approaches that focus solely on user-item

---

This work is supported in part by the National Science Foundation under the awards CNS-0747247, CCF-0914946, CNS-1314229 and NSF #0643532 and by an NSA Science of Security Lablet grant at North Carolina State University. We would also like to thank the anonymous reviewers for their valuable feedback. Authors addresses: Euijin Choo, email: echoo@ncsu.edu; Ting Yu, email: tyu@ncsu.edu; tyu@qf.org.qa; Min Chi, email: mchi@ncsu.edu; Yan Sun, email: yansun@ele.uri.edu.

EC'14, June 8–12, 2014, Stanford, CA, USA.

http://dx.doi.org/10.1145/2600057.2602906

relationships. The key challenge when applying social systems, however, is to determine which connections should be employed. Explicit social networks may often be ineffective because established friendships are frequently irrelevant to personal preferences. Two individuals may friend one-another on Facebook because they have taken the same math class, but that relationship has no bearing on their taste in film. Moreover, many influential application platforms that would benefit from social recommendations such as Amazon, CNet, and Urbanspoon, do not permit users to state explicit social relationships, and it is difficult, if not impossible, to import relationships from other sources; and privacy concern arises when we integrate commercial systems with social networks.

Our goal in this research is to address these limitations by determining how we can identify meaningful relationships through user connections on existing, non-social, platforms. We have observed that many influential recommender systems such as Amazon and Yelp permit users both to post item reviews and to respond to prior reviews. Users may reply to a review for a variety of reasons (e.g. signifying agreement or disagreement or seeking additional information). From these connections we can extract implicit relationships which can in turn be extended to implicit communities. In this paper we will focus on extracting relationships of this type from the Amazon platform.

In this work we consider the act of replying to a user to be an *interaction* between two users: the *reviewer* who authored the initial review; and the *commenter* who authors the reply. While it is initially tempting to treat each such interaction as a sign of a meaningful relationship, many such replies are likely to occur by chance. Users will frequently browse for an item of interest, read a review, and post a simple comment without considering the author of the original post. Thus most such interactions can be modeled as a random process. It is necessary to filter out these random interactions in order to provide high-quality personalized guidance. Our goal is thus to develop a rigorous and systematic mechanism to identify significant and non-random connections between users based upon their reviewer-commenter interactions which can then be used to improve the existing recommender systems.

There are several fundamental questions that need to be investigated in this work: (1) Do significant non-random connections exist in practice, and can we separate them from the meaningless relationships? (2) Given a pair of users, how can we quantify the strength of their relationship if any and identify any meaningful connections? (3) Once implicit communities are discovered, will they have any relevance to recommender systems or, indeed, any relationship to the individual users' preferences?

We begin by proposing a probabilistic approach to measure the strength and randomness of reviewer-commenter interactions. We will then apply this measure to show that strong non-random relationships exist between Amazon users. We call these extracted connections *implicit relationships* as opposed to the *explicit social relationships* found in typical social networks such as Facebook. We will then show that such implicit relationships can provide a direct indication of mutual interests, and can thus be used to substantially improve recommendation accuracy over existing collaborative filtering techniques that only take into account user reviews of common items.

Our main contributions are summarized as follows.

- We develop a random graph model that represents users as vertices and reviewer-commenter interactions as edges. The graph allows us to distinguish between random and non-random interactions and to study the pattern of real user interactions. If all user interactions occur by chance then real user data will closely match the random graph. If, however, the user data deviates substantially from this model then meaningful relationships are likely to be found.
- We develop a quick global indicator that can be used to highlight when, and to what extent, interactions among users deviate from the random model. We observe that interactions in *subjective* item categories (e.g. book and movie reviews) tend to deviate substantially from the random model while *objective* categories (e.g. electronics and tools) tend to have more random interactions.
- We propose a probabilistic mechanism to quantify the strength of connections between a pair of users based upon the random model which we use to construct a series of relationship graphs.

— We apply existing social recommender algorithms using the extracted relationship graphs and show that the resulting recommendations are more accurate than those produced by collaborative filtering methods.
— Our probabilistic approach allows to adjust trade-off between recommendation accuracy and coverage. We show that communities defined by different strength can be used for hybrid recommender systems to improve both accuracy and coverage over collaborative filtering methods.

The remainder of this paper is organized as follows. In Section 2, we review related work. Section 3 presents an abstract model of recommender systems, and introduces basic concepts and notations used through this paper. In Section 4, we describe how we collected data for our experiments and present a few characteristics of this dataset. Section 5 offers results to test the randomness of each user's interaction in Amazon, our definition of a community, and discovered communities in dataset. In Section 6, we delineate how to utilize discovered communities for the purpose of recommendation and discuss experimental results. Finally, Section 7 concludes the paper.

2. RELATED WORK

Many recommender systems have been proposed in the literature [Adomavicius and Tuzhilin 2005; Konstan 2004; Pazzani and Billsus 2007; Resnick and Varian 1997; Ricci et al. 2011; Schafer et al. 2007]. They fall into two general categories: content-based recommendations, and collaborative filtering [Adomavicius and Tuzhilin 2005; Funakoshi and Ohguro 2000; Herlocker et al. 2004; Linden et al. 2003; Sarwar et al. 2001; Schafer et al. 2007]. Content-based approaches [Funakoshi and Ohguro 2000; Mooney and Roy 2000; Pazzani and Billsus 2007] recommend an item by comparing it to items in the user's personal history. Hulu, for example, populates a list of "You may also like..." shows by identifying actors, directors, and genres that the user has seen before.

Collaborative filtering approaches, on the other hand, seek to identify users that share common items and then provide recommendations based upon these matching peers [Herlocker et al. 2004; Huang et al. 2007; Schafer et al. 2007]. The key observation is that two users who have previously selected a number of items in common are likely to purchase similar items in the future. Amazon, for example, uses collaborative filtering to populate the list of items entitled: "Customers Who Bought Items in Your Recent History Also Bought".

While both approaches have been used widely, they both suffer from data sparsity and cold-start problems [He and Chu 2010; Xu et al. 2012]. The former arises from the fact that users have often selected and commented on a small number of items thus providing little information for matching algorithms to go on. The latter problem arises from the fact that new users have no history and thus we have a small amount of data on which to base our recommendations. Social recommender systems have been put forward as a candidate solution to these problems based upon the belief that advice from friends plays an important role in decision making [Arazy et al. 2009; Bonhard and Sasse 2006; DuBois et al. 2009; He and Chu 2010; Ma et al. 2013, 2011].

While social recommender systems have improved accuracy and better handled the data sparsity problem [Adomavicius and Tuzhilin 2005], explicit social information is not always available in recommender environments. Connecting recommender systems with social networks, however, raises privacy issues surrounding personal information that is neither relevant to recommendations nor should be disclosed [Machanavajjhala et al. 2011]. Furthermore, there is no guarantee that friends always share similar interests [Ma et al. 2011], and the value of this information is degraded further in systems that combine near and distant relationships [He and Chu 2010]. This may explain why it has been shown that employing explicit social information in recommender systems for news articles actually lowers the prediction accuracy [Shmueli et al. 2012].

With that in mind we propose a method to identify implicit user relationships in recommender systems by analyzing two different type of user actions: posting reviews, and replying to them. We will show that the use of these implicit relationships, both immediate and distant, improves the accuracy recommendation systems over existing collaborative-filtering approaches.

While the importance of social recommendations has been studied before, to the best of our knowledge no prior work has been done on the use of implicit relationships arising from direct interaction in recommender systems. Several papers take into account different types of interactions in news sites [Lee et al. 2012; Leprovost et al. 2012; Schuth et al. 2007]. However, these studies focused primarily on extracting the discussion structure (e.g. word counts and sentence lengths) and location of replies [Schuth et al. 2007]; or on evaluating the content of reviews and replies [Lee et al. 2012; Leprovost et al. 2012; Schuth et al. 2007]. None of them have examined the relationships between users. Rather they focus on novel techniques to combine content-based and collaborative filtering approaches, and consequently they inherit the limitations of traditional approaches. Our implicit relationship structures, by contrast, help to improve upon existing systems and address these limitations. More importantly, the use of implicit relationships allows us to control the trade-offs between improving recommendation accuracy and sensitivity to these general limitations.

3. RECOMMENDER SYSTEM

A recommender system analyzes large datasets to help users make personal decisions. It applies users' implicit or explicit preferences to filter the available items, which may include digital contents (e.g., webpages or video clips on Youtube), products (e.g., kitchenware), or people (e.g., potential friends in social networks) [Ricci et al. 2011].

In general, there are two components in a recommender system: users and items. Users consume and evaluate items, and the system recommends items to users. Both users and items have a set of system-specific attributes relevant to recommendations. Hulu and Last.fm, for example, classify items by artist and genre while other systems track demographic information and purchasing history.

In this work we focus on two different user actions: reviewing items and commenting on reviews. *Reviewing* occurs when a user posts her evaluation/opinion about an item to the system. *Commenting* occurs when a user replies to an prior reply or review. Both reviews and comments may take a variety of forms including assigning star ratings, voting, and writing text. For example, Amazon allows users to write text reviews along with star ratings, to write text comments, and to vote helpfulness of the review; while Urbanspoon does not allow text comments but allow helpfulness votes on reviews. A user may thus take one of two roles in any relationship: *reviewer* and *commenter*.

Every time that a commenter replies to an existing review we consider it to be an interaction with the reviewer. We define implicit relationships based upon these reviewer-commenter interactions in Section 5. Note that in a recommender system, user u_1 can leave a comment, say c_1, on an existing comment, say c_2 by another user u_2, which generates threads of comments. In such a case, the interaction would be between u_1 and u_2. A user can also post multiple comments on the same review/comment in a system. Such actions often occur when a user wants to have a more detailed discussion on the item, or seek additional information.

Let us assume that two users like one item in common by chance so that they had a lot of discussions on the item (i.e., generating threads of comments, or post multiple comments to one review). Although they had a lot of interactions with each other, it does not mean they have a meaningful relationship (e.g., socially connected or share the same item preference in general) with each other. Instead, those users are only interested in the specific item. In the following, we thus only consider the interaction between a reviewer and a commenter, not between a commenter and another commenter; and we count multiple comments on the identical review/comment as a single interaction.

4. DATASET

This section explains the dataset used in this study. Specifically, we describe how we collected data for our experiments, and present a few characteristics of this dataset. We implemented a data crawler with Perl. Amazon is one of the most popular recommendation platforms with worldwide users, so we collected data from Amazon and conducted experiments over the data. The crawler crawled information about items, reviews, and comments. Information about an item includes an item identifier and the category of the item as assigned by Amazon (e.g., Music, Movie). Information about a review includes its target item (i.e., the item about which the review is), the user identifier of the reviewer, and its rating ranging from 1 to 5. Information about a comment includes the user identifier of the commenter, and a target review (i.e., the review about which the comment is).

Table I. Dataset

Category	#items	#reviews	#comments	#reviewers	#commenters
Books	116,044	620,131	533,816	70,784	164,209
Movie	48,212	646,675	201,814	273,088	72,548
Electronics	35,992	542,085	128,876	424,519	72,636
(Home&Kitchen) Tools	22,019	229,794	32,489	151,642	21,977

To collect different item categories, we employed a breadth-first search of Amazon starting from a random item in each category. In the item's page, we collected information about the item, all reviews for the item, and all comments for the reviews. Then, the crawler proceeded to pages of all users crawled (i.e., all reviewers and commenters). On each user's page, we can find all of his reviews and their target items. The crawler thus proceeded to pages of those items to do the same breadth-first crawling. By doing so, the crawler can grab items to which at least one review is posted.

We will analyze different characteristics of each item category in sections 5 and 6. Concretely, we will show distinguishing characteristics of two different types of item categories: subjective and objective item categories. To do so, we performed experiments on two subjective categories (Books and Movie) whose reviews tend to be subjective and two objective categories (Electronics and Tools) whose reviews tend to be objective. The details of the dataset are summarized in Table.I

5. REVEALING IMPLICIT COMMUNITIES IN RECOMMENDER SYSTEMS

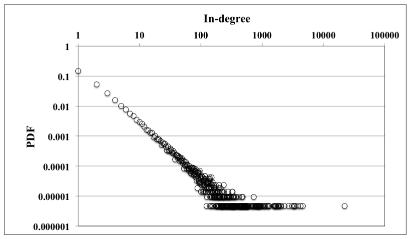
Our goal is to find implicit yet meaningful relationships among users in a recommender system. To do so, we first explain a typical model of user interactions in the system in Section 5.1. Then, based on the model, we present a global measure to study patterns of the interactions in Amazon as an example of real applications in Section 5.2. To be specific, we will compare interactions among real users with the typical model of interactions to capture the difference between random/non-random interactions. Finally, in Section 5.3, we explain how we define a *user relationship* and a *community*, and present implicit communities found in Amazon.

5.1. Modeling Interactions in Recommender Systems

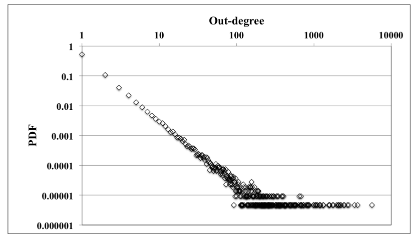
Let us present a typical scenario about a user's activity in a system without explicit social relationships (e.g., Amazon). Let us assume that user u has interest in item p, and u found user v's review about p interesting. u wants to have discussion with v about p, so u posts a comment to v's review. Although u left a comment on v's review, from this instance it does not indicate that there is any special connection between u and v. Instead, it is more likely that this interaction happens in a random manner: u does not know v before and does not specifically look for v's reviews either. u simply *bumps into* v's review by chance during browsing. Therefore, a typical interaction between a commenter and a reviewer can be modeled as a random process. Then, intuitively, a special connection exists between a pair of users only if their interactions happen beyond random acts.

The next question is then how to determine whether two users have non-random interactions. A compelling measure is to look at the number of interactions between them. Intuitively, the more interactions between them, the less likely they happen randomly. However, purely looking at the number of interactions is not sufficient as it affected by a lot of factors. For example, some user u may have a tendency to engage in discussions with others. So the mere fact that u has many (say 10) interactions with a reviewer does not necessary mean the reviewer is anything special to u (as it is possible u may have similar number of interactions with many other reviewers in a random fashion). On the other hand, if u rarely issues comments to others, then it may indicate strong and some unusual connections between u and v if u comments on 10 of v's reviews.

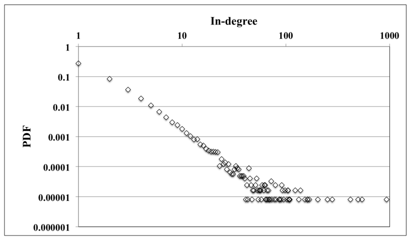
To distinguish random/non-random interactions, we first present a model to capture random interactions of users. We represent users and their interactions as a directed multigraph $G = (U, E)$ in which U represents users (vertices) and E represents interactions (edges). Since an interaction is defined as an action for a commenter to post a comment on a reviewer's specific review, an interaction has a direction from a commenter to a reviewer.



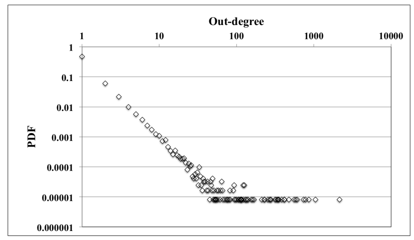
(a) Books in-degree distribution



(b) Books out-degree distribution



(c) Electronics in-degree distribution

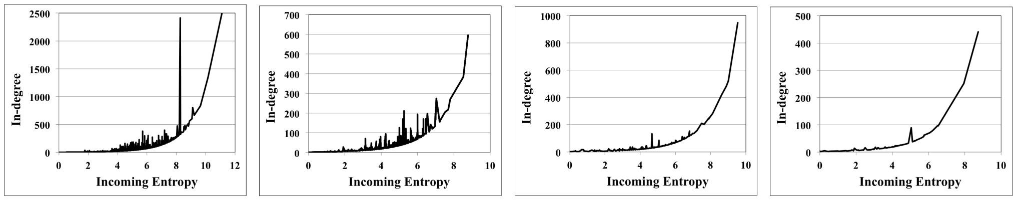


(d) Electronics out-degree distribution

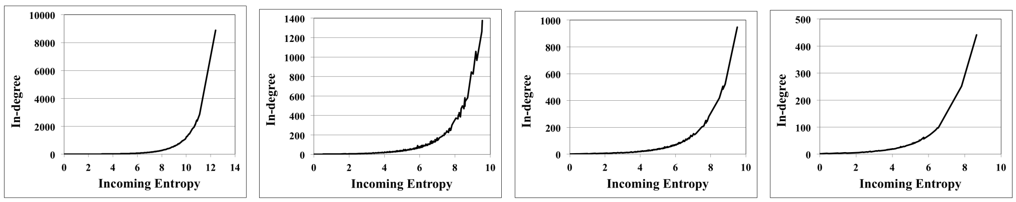
Fig. 1. In-degree and out-degree distributions of each user in Books and Electronics

Each edge $\overrightarrow{e_{uv}}$ is a 2-tuple (u, v), where u is a commenter and v is a reviewer. Since we count multiple comments from the same user u on the same review of user v as a single interaction from u to v, the number of edges from u to v is in fact the number of different reviews of v on which u commented. A user as a commenter in a graph has outgoing edges and a user as a reviewer has incoming edges. An *out-degree* of commenter u is the total number of edges from u to other users and an *in-degree* of reviewer v is the total number of edges from other users to v. A *total-degree* of a user is the sum of out-degree and in-degree of the user. Since the in-degree of v is the number of comments from other users to v, it essentially indicates how popular v is to get comments (i.e., how interesting v's reviews tend to be); while the out-degree of u is the number of comments from u to other users, which indicates the tendency of u to write a comment (i.e., how much u is willing to comment). Fig.1 shows the in/out-degree distributions of users in each category. The X-axis represents in/out degree and the Y-axis represents corresponding probabilities. The in/out-degree distributions in different categories were similar, so we only present the distributions in two categories.

In general, the in-degree and the out-degree represent a user's personal tendencies as a commenter and a reviewer, respectively. For example, some user may write a lot of reviews and get a lot of comments by many (high in-degree); while some user seldom writes reviews, but posts a lot of comments on the existing reviews (high out-degree). To characterize those different nature of individuals, it is needed to examine a user's tendencies as both a commenter and a reviewer separately. We model such user's different tendency as an *outgoing probability* and an *incoming probability*. The *outgoing probability* of u is the probability that u generates outgoing edges and an *incoming probability* of v is the probability that v gets incoming edges. If all the interactions in a recommender system are not guided by any special relationship between users, interactions from u to v happen randomly depending on u's outgoing probability and v's incoming probability. Then, we can represent all users' interactions as a random graph in which edges (i.e., interactions) are created following the incoming/outgoing probability of each user. Then, the total number of all edges and each user's degree distribution in the random graph is the same as the original graph; but edges are randomly generated and the number of edges between each pair of users will be different from G [Newman et al. 2001].



(a) Books real (b) Movie real (c) Electronics real (d) Tools real



(e) Books random (f) Movie random (g) Electronics random (h) Tools random

Fig. 2. Incoming entropy and in-degree of each user in each category

5.2. Randomness of Interactions in Amazon

To study interactions among real users, we present our observations on Amazon dataset and compare it with the random model presented in Section 5.1.

We construct a random graph by generating edges randomly with the given incoming/outgoing probabilities of users in the real dataset. Then, we compare a graph in the real dataset with the corresponding random graph to see how interactions are different from random interactions. By doing so, we will show that there exist implicit communities in Amazon.

In information theory, entropy is a measure of unpredictability (randomness) [Dodis and Smith 2005]. We thus employ entropy as a measure to assess the randomness of interactions among users.

Let X_u denote the event that an edge is formed from user u to other users and x_{uv} denote the event that an edge is formed from commenter u to reviewer v. Then, X_u's entropy $H(X_u)$ for outgoing edges, so called *outgoing entropy*, is given as follows:

$$H(X_u) = -\sum_{v \in U_u} P(x_{uv}) log_2(P(x_{uv})), \tag{1}$$

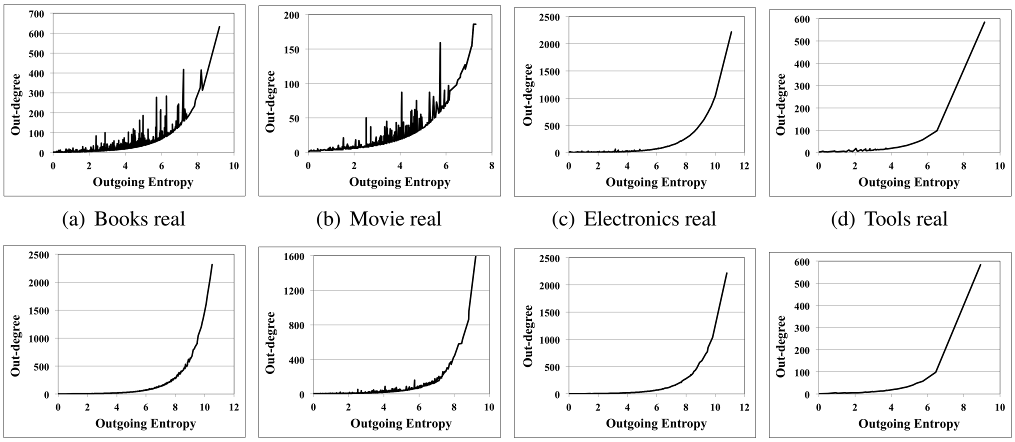
where $H(X_u)$ is the entropy value, $P(x_{uv})$ is the probability that x_{uv} occurs, and U_u is the set of users who have interacted with user u.

Let Y_v denote the event that an edge is formed from other users to v and y_{uv} denote the event that an edge is formed from commenter u to reviewer v. Then, Y_v's entropy $H(Y_v)$ for incoming edges, so called *incoming entropy*, is given as follows:

$$H(Y_v) = -\sum_{u \in U_v} P(y_{uv}) log_2(P(y_{uv})), \tag{2}$$

where $H(Y_v)$ is the entropy value, $P(y_{uv})$ is the probability that y_{uv} occurs, and U_v is the set of users who have interacted with user v.

Fig.2 and Fig.3 show users' incoming/outgoing entropy and in-degree/out-degree in real/random dataset of four categories - Books, Movie, Electronics, and Tools. Random dataset was built based on the random model in Section 5.1. In order to visualize a relationship between entropies and corresponding degrees clearly, values were sorted by entropy. The X-axis represents sorted incoming/outgoing entropies and the Y-axis represents corresponding in/out degrees.



(a) Books real (b) Movie real (c) Electronics real (d) Tools real

(e) Books random (f) Movie random (g) Electronics random (h) Tools random

Fig. 3. Outgoing entropy and out-degree of each user in each category

Usually, if an event (i.e., edge) is randomly generated (i.e., interactions are randomly distributed), a user's entropy and degree should be proportional by definition of the entropy (i.e., Eq.1 and Eq.2). Indeed, we can see that in the random model dataset, the in-degrees and the out-degrees were almost proportional (i.e., smooth line graph) to the corresponding incoming entropies and outgoing entropies, as shown in Fig.2 and Fig.3.

In the real dataset, on the other hand, the figure of each category shows a different characteristic. Concretely, in/out degree was out of proportion (i.e., a lot of spikes) to the corresponding entropy values in two subjective categories: Books and Movie. Those spikes mean that the entropy is low, even when the degree is high. That is, a large portion of interactions of those users are focused with a few users, resulting in low entropies given high degree. Hence, it suggests that the interactions of such users with spikes are not random. Note that the general shape of the graph without spikes are similar to that of random model. This lends support to our assumption that user interactions occur randomly in general, but some users have beyond-random interactions with each other.

On the other hand, the figures of two objective categories, Electronics and Tools, were very close to those of the random model dataset as shown in Fig.2 and Fig.3. One possible explanation is that people usually share opinions and interact a lot throughout various items in subjective categories (e.g., books and movie), whereas people interact to grab information about specific sets of items in objective categories (e.g., electronics and tools). In other words, people's interactions about objective items are rather limited to a small set of items of interest than various items; therefore, their interactions are similar to a random model.

From these results, we can see that relationships between users in different categories may show different characteristics. For example, many relationships in certain categories are random; while there may exist non-random user relationships in certain categories, so that interactions between users occur because of their relationships, instead of purely randomly. We will show these different characteristics actually affect the accuracy of different recommendation algorithms in Section. 6.2.

In the following sections, we will define a *user relationship* and a *community* in recommender systems, and describe implicit communities found in Amazon based on our definitions.

5.3. User Relationship

To define a *user relationship*, let us first give an example scenario. Suppose that u's interest is similar to v's, u thinks v's reviews are always interesting, and u posts comments to many of v's reviews. Then, we may say that u *recognizes* v in a certain way, so that u is likely to be influenced by v.

With the consideration of a random model discussed in Section 5.1 and a scenario above, let us define a *user relationship*. Intuitively, if there are a large number of interactions between two users compared with the number of expected interactions in the random model, we may assume that those two users are somewhat related. Based on the assumption, we denote u is *related* to v, when the number of u's comments on v's different reviews lies outside the range of the expected number in the random model.

Suppose that there is graph G with the total number of all edges n, and u and v are vertices (users) on G. There are ω interactions from u to v, which means the number of edges from u to v is ω. Meanwhile, u's out-degree is O and v's in-degree is I. So, if they are not related, the chance for an edge from u to v to form is $\rho_{uv} = (O/n) * (I/n)$.

Given the mean $\overline{X_{uv}}$ and the standard deviation σ, confidence intervals are often computed using z scores [Sprinthall and Fisk 1990]. To compute τ% confidence interval, α (= 1-$x/100$) and z is defined. For example, α is 0.05 and z is 1.96 for the 95% confidence interval. For each τ, the upper and lower bounds of τ are $\overline{X_{uv}} \pm z_\tau * \sigma/\sqrt{n}$, where z_τ is a z-score for τ confidence interval.

If the probability ρ_{uv} is larger than the upper bound, it indicates that u and v have less interactions than the expected one, which is also common in a random case; because if u and v do not share any interest, they will not interact with each other. Accordingly, we only take into consideration the case that the probability is less than the lower bound.

A formal definition of user relationship is given as follows.

Definition 5.1. u is <u>*related*</u> to v with τ confidence interval, if the following condition holds:

$$\rho_{uv} < \theta_{uv},$$

where ρ_{uv} is the probability for edge $\overrightarrow{e_{uv}}$ to form in graph G and θ_{uv} is the lower bound of given confidence interval τ (i.e., $\overline{X_{uv}} - z_\tau * \sigma/\sqrt{n}$).

Given Definition.5.1, we quantify the strength of relationship in two ways. The larger difference between ρ_{uv} and θ_{uv}, the less likely the interactions between u and v happen randomly, and thus the more likely a strong relationship exists between u and v, which, in turn, the more influences v has on u. The strength can thus be defined with the difference between ρ_{uv} and θ_{uv} as follows.

Definition 5.2. The strength Δ^1_{uv} of user relationship between u and v is

$$\Delta^1_{uv} = |\rho_{uv} - \theta_{uv}|,$$

where ρ_{uv} is the probability for edge $\overrightarrow{e_{uv}}$ to form in graph G and θ_{uv} is the bound, $\overline{X_{uv}} - z_\tau * \sigma/\sqrt{n}$.

On the other hand, the wider confidence interval τ with which the relationship is defined, the more likely a strong relationship exists between u and v by Definition 5.1. The strength can thus be defined with the confidence interval τ as follows.

Definition 5.3. The strength Δ^2_{uv} of user relationship between u and v is

$$\Delta^2_{uv} = \tau_{uv},$$

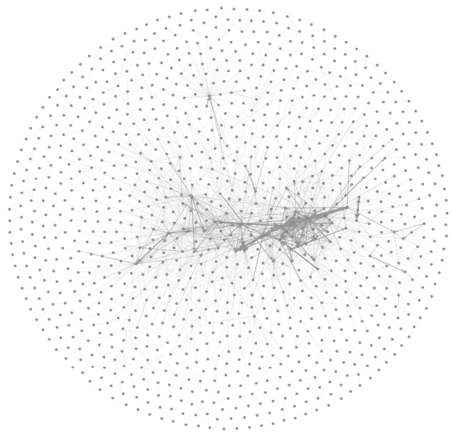
where τ is confidence interval with which the relationship between u and v is defined.

The user relationships can be extended to communities in which users are related to another while constructing user relationship graphs. The communities are defined with the confidence interval used in the definition of user relationships as follows.

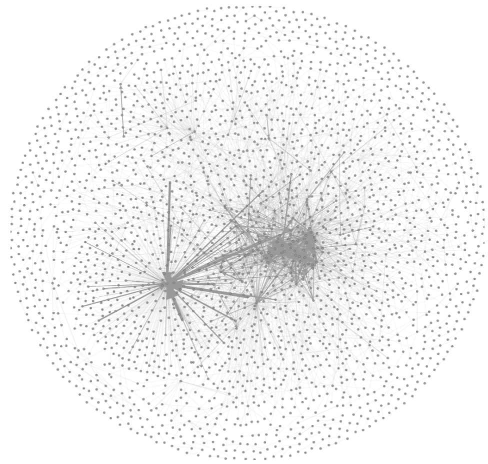
Definition 5.4. edge $\overrightarrow{e_{uv}}$ (in turn, user u and v) belongs to <u>*τ% community*</u>, if the following condition holds:

$$\rho_{uv} < \theta_{uv}$$

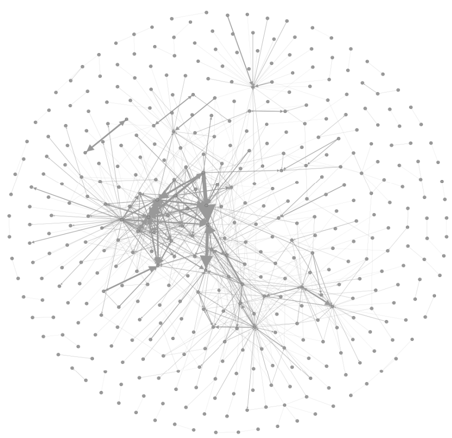
where ρ_{uv} is the probability for edge $\overrightarrow{e_{uv}}$ to form, and θ_{uv} is the bound of τ% confidence interval.



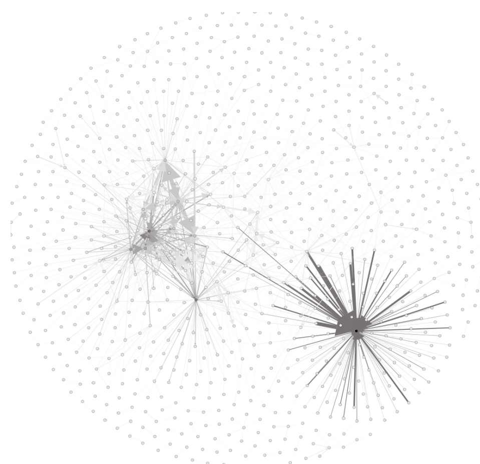
(a) Movie 90% community



(b) Book 90% community



(c) Movie 99% community



(d) Book 99% community

Fig. 4. Discovered communities by a proposed approach

5.4. Revealing Implicit Communities in Recommender Systems

In this section, we present implicit communities revealed in our dataset from Amazon.

Fig.4 represents discovered $\tau\%$ communities from two categories - Books and Movie. The thickness of an edge represents the number of interactions. Accordingly, the thicker edge $\overrightarrow{e_{uv}}$ is, the more interactions from u to v exist. Relationships with thicker edges are thus stronger than relationships with thinner edges. The darkness of vertices represents the total degree of the vertices.

As shown in Fig.4, there are two patterns of sub-communities. One is a so called *following community* in which there are a few influential users (reviewers) who get a lot of comments from many commenters, but do not leave many comments to the commenters. We refer to the relationship between influential users and commenters as a *following relationship*, because it is similar to the relationship between followers and followees on Twitter. Note that the influential user (followee) does not leave many comments to followers. Consequently, it is not necessary that an influential user's opinion depends on the followers' ones. Another pattern of sub-communities is a so called *strongly correlated community*. Compared with the one-directional interactions in the following communities, interactions in strongly correlated communities are bi-directional. In this case, users are mutually related to each other, so that the users have an influence on each other.

Recall that the strength of a user relationship is defined by τ (Definition 5.3). Hence, relationships in Fig.4(c) and Fig.4(d) are stronger than those only in Fig.4(a) and Fig.4(b). As delineated in Section 5.3, a stronger relationship between users means that there are more interactions between the users than the expected. Note that multiple comments to the identical review are counted as a single interaction. Also, a commenter's intention behind comments on a review is essentially to discuss the item of the interest. Many interactions between a commenter and a reviewer thus indicate that they share interests in many items. Hence, we argue that users who have stronger relationships would have more influences on each other than weaker relationships and those whose opinions will be more helpful to improve recommendation accuracy, which will be shown in Section 6.2.

Also, following communities tend to remain with larger $\tau\%$ community as shown in Fig. 4. These results mean that there is a tendency for following communities to include stronger relationships than a strongly correlated community. As mentioned above, it is not necessary that followee's opinion depends on the followers' ones. We thus argue that considering directions of discovered implicit relationships will be more helpful than ignoring directions, which will be also shown in Section 6.2.

6. RECOMMENDATION

In this section, we delineate how to utilize discovered implicit communities for the purpose of recommendation and evaluate the significance of the communities. The main idea is to treat users in the community as social relationships, i.e., friends.

6.1. Recommendation with Discovered Communities

Our goal is to show that discovered implicit communities are relevant to item preference and they can be incorporated into social recommender systems to improve recommendation accuracy. To do so, we employed commonly used social recommender models [He and Chu 2010; Ma et al. 2011; Shmueli et al. 2012], details of which are given below.

In previous social recommender models, a predicted rating for an item is often computed as an aggregation of friends' ratings to the item [Ma et al. 2011]. One way to aggregate is to use the average of friends' ratings, referred as *an average-based approach*, based on assumption that a user's taste should be close to the average tastes of its friends. The formal definition of user u's predicted rating for item i ($r_{u,i}$) by averaging friends' ratings is as follows.

$$r_{u,i} = \frac{1}{n} \sum_{v \in F} r_{v,i},$$

where n is the number of u's friends, v is a user in the set F of u's friends, and $r_{v,i}$ is v's rating for the item i.

However, user u may have a lot of friends with different tastes, so that their ratings are not meaningful to predict u's ratings. Another way is to treat each friend differently according to how similar u and v are, referred as *a similarity-based approach*. The formal definition of user u's predicted rating for item i ($r_{u,i}$) by employing friends' similarity is as follows.

$$r_{u,i} = k * \sum_{v \in F} sim(u, v) * r_{v,i},$$

where sim(u,v) is the similarity between u and v and k is a normalizing factor defined as

$$k = \frac{1}{\sum_{v \in F} |sim(u, v)|}$$

One of common methods to compute the similarity is Vector Cosine Similarity (VCS) whose definition is as follows.

$$sim(u, v) = \frac{\sum_{i \in I_{uv}} r_{u,i} * r_{v,i}}{\sqrt{\sum_{i \in I_{uv}} r_{u,i}^2} \sqrt{\sum_{i \in I_{uv}} r_{v,i}^2}},$$

where I_{uv} is the set of items u and v have in common.

There are a variety of ways to define who are one's "friends" for the purpose of recommendation [He and Chu 2010]. In other words, we need to determine whose ratings should be included when predicting a user's rating. One option is to only consider *immediate users* (i.e., one hop away in a relationship graph) and the other is to consider both immediate and *distant users* (i.e., multiple hops away in a relationship graph). Relationships in our approach also have directions from commenter u to reviewer v as mentioned in Section 1 and Section 5. Thus, we have the options to either use directional information or to ignore it. When considering the direction, we include reviewer v's ratings to predict commenter u's rating but not vice versa. This is based on our assumption that while the reviewer v's opinions are more likely to have influence on commenter u, it is less clear that commenter u's opinion would have influence on reviewer v. Alternatively, we can ignore directional information and when predicting a reviewer v's rating, we use commenter u's ratings as well.

When considering distant users, we may further apply community detection techniques to $\tau\%$ community. For example, we may use users' ratings as long as there is a path in a relationship graph from/to a user; we may consider k-hop distant users, so that we use users' ratings, if they are k hops away from/to a user;or, we may apply clustering techniques to filter out users who are too far away in a relationship graph. The techniques used in this paper will be discussed in Section 6.2.

6.2. Comparison with Collaborative Filtering Methods

In this section, we evaluate the significance of the discovered communities. The significance will be validated by analyzing recommendation accuracy and how many users can get benefit from the proposed approach. As discussed before, there are a variety of ways to determine whose ratings will be used to predict a user's ratings. Let F_u denote the set of users whose ratings will be used to predict user u's ratings. We introduce 3 approaches - *directed*, *undirected*, and *connected* - to determine F_u. We compared those approaches and two representative existing popular collaborative filtering methods: the memory-based nearest neighbor (NN) [Sarwar et al. 2001], and the latent factor model-based (LFM) [Koren et al. 2009] algorithms. NN finds similar users, whereas LFM discovers factors to predict.

For the *directed*, we considered the direction of interaction. When there is an edge from commenter u to reviewer v in a relationship graph, we used v's ratings to predict u's ratings but not the other way. In other words, v belongs to F_u but u does not belong to F_v. For the *undirected*, we ignored directions so that we used a reviewer's rating to predict a commenter's ratings as well as a commenter's ratings to predict a reviewer's ratings. That is, v belongs to F_u and u belongs to F_v.

While for both directed and undirected approaches, we considered only immediate users, for *connected approaches*, we considered both immediate and distant users as long as they are connected in our implicit community. Furthermore, in this approach, we ignored directional information. In other words, all users who have a path in the relationship graph from/to u belong to F_u.

When we combine the three categories on how to define F_u with two aggregation methods: average-based or similarity-based, we have our three average-based approaches: directed average (DA), undirected average (UA), connected average (CA) and three similarity-based approaches: directed similarity, undirected similarity, and connected similarity. Here we used vector cosine similarity to compute a similarity for our 3 similarity-based approaches.

First, we compared our three average-based approaches with three similarity-based approaches. Our results showed that the average-based approaches out-performed the similarity-based ones. We argue it is because similarity with other users cannot be measured without previous rating. That is, if user u does not have any item in common with other users, similarity-based approaches can neither use u's rating to predict other's nor give recommendations to u. On the other hand, the average-based approaches can take u's rating to predict all related users, that is, the average-based approaches can use more information than the similarity-based ones, which resulted in the better accuracy and coverage. Also, commenters without previous ratings can not get recommendation by a similarity-based approach, because similarity cannot be measured; but the average-based approach can give recommendation to commenters without previous rating using related users' ratings. Therefore, in the following, we only present the results of three average-based approaches. More specifically, we compared five approaches: our three average-based approaches, directed average (DA), undirected average (UA), and connected average (CA) and two state of the art collaborative methods: NN and LFM, in terms of both prediction accuracy and coverage.

6.2.1. Prediction Accuracy. A recommender system predicts each user's rating for an item to judge whether the user likes/dislikes the item. We thus predicted a user's rating and compared the predicted rating with the real rating of the user. The rating ranges from 1 to 5. The prediction accuracy is measured by mean absolute error (MAE), which is defined as follows.

$$MAE = \frac{\sum_{u,i} |r_{u,i} - r'_{u,i}|}{n},$$

where $r_{u,i}$ is user u's rating about item i, $r'_{u,i}$ is an estimated value for u's rating about item i, and n is the number of items that u have ratings. The smaller MAE is, the more accurate a prediction is. Note that accuracy is measured throughout all items in the dataset.

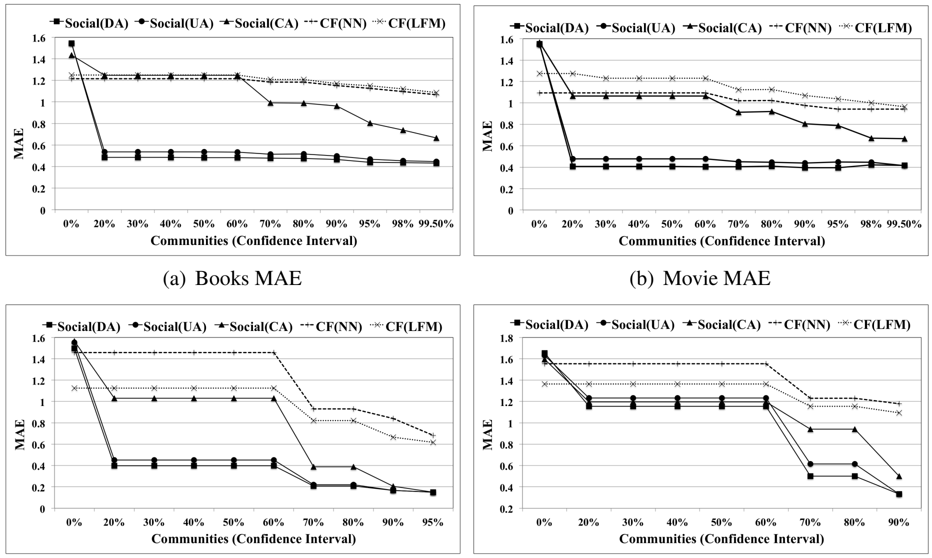
We compared the prediction accuracy of our 3 approaches using implicit communities and two collaborative filtering methods across four categories: Books, Movie, Electronics, and Tools. Fig.5 represents MAE of those 5 approaches across various $\tau\%$ confidence communities. The X-axis represents each confidence community as defined in Section 5 and the Y-axis represents the average of MAE of predicted ratings for the community users. Recall that interactions between users in Electronics and Tools are more similar to the random model. Hence, there existed only a few users in a larger $\tau\%$ community so that users who are in larger $\tau\%$ communities (i.e., 99.5%, 98% in Electronics, 99.5%, 98%, 95% in Tools) could not get predictions with a limited amount of information.

As shown in Fig.5, the directed approach (DA) performed better (lower MAE) than the two undirected ones: undirected (UA) and connected (CA) approaches. This result suggests that a commenter's opinion may not have much influence on a reviewer's opinion; thus when predicting a reviewer's ratings, it is more beneficial to exclude commenters' ratings.

Since it might limit the number of users who could get predictions when considering only immediate users, we employed the connected approach. Although more users' ratings can be used to predict a user's ratings, it increases the possibility to include more weakly related users. Consequently, the predicted ratings with CA were less accurate (larger MAE) than DA and UA. Note that MAE dropped dramatically between 60% and 70% communities with the connected approach. It occurred because many *weak relationships* started to be included when we used the τ smaller than or equal to 60%; Note that weak relationships mean that two users' would have less influences on each other. Thus, a user's rating may not be close to the average of his/her weakly related users' ratings. Hence, MAE will be larger as we include more weakly related users.

Also, there was a big drop between 60% and 70% communities with the directed or undirected approaches in Electronics and Tools, while there was no such a big drop in Books and Movie. It is because interactions in Electronics and Tools were more similar to the random model and relationships were thus relatively weak; while relationships in Books and Movie were relatively strong. Consequently, the accuracy will be worse as many weak relationships are considered. As shown in Fig.5, using 0 % community (i.e., users are considered, once they have at least one interaction) make prediction accuracy worse than NN or LFM.

In many papers, LFM was shown to be more accurate than NN [Hofmann 2004; Koren 2010; Ma et al. 2011]. Indeed, LFM performed better for community users in Electronics and Tools where interactions follow the random model more closely as shown in Fig.5. However, for community



(a) Books MAE

(b) Movie MAE

(c) Electronics MAE

(d) Tools MAE

Fig. 5. MAE in each category

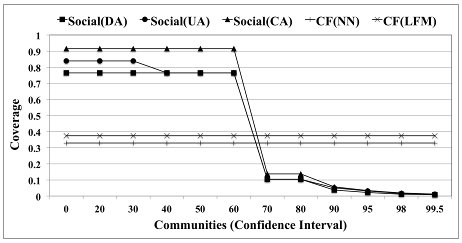
users in Books and Movie where interactions do not follow the random model, NN provides almost similar or better prediction accuracy than LFM as shown in Fig.5. We argue this is because NN considers user similarity relationships and the taste of related users in Books and Movie are more similar to each other. In other words, considering user similarity relationships (NN) performed better than considering latent factors (LFM).

In short, we showed that when incorporating our implicit communities, all three average-based approaches out-performed the two state of the art approaches: NN and LFM across all four categories on predicting users' ratings.

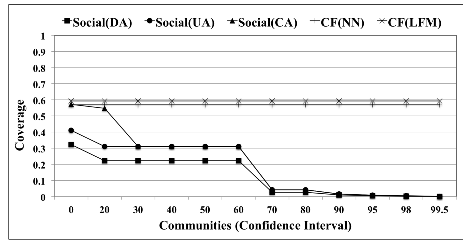
6.2.2. Coverage. While we have shown that recommendation using discovered implicit communities can improve prediction accuracy on a recommender system, it is also important for us to show how many users can potentially benefit from those communities. Therefore, we measured the percentage of users in each category who can get predictions from 5 approaches.

Fig.6 shows the coverage of 5 approaches from Books, Movie, Electronics, and Tools. Recall that relationships in a larger $\tau\%$ community (i.e., $99\% > 98\% >$..) are stronger than those only in a smaller $\tau\%$ community; and there were fewer users in a larger $\tau\%$ community as shown in Fig.4, which means many users in the system may not get predictions; whereas there are more users in a smaller $\tau\%$ community, which means more users in the system can get predictions.

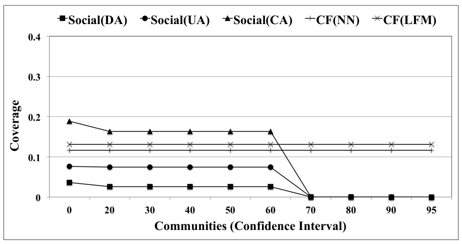
As mentioned before, there were more strongly related users in Books and Movie, compared to Electronics and Tools. That is, more users could get predictions in Books and Movie, compared to Electronics and Tools. As shown in Fig.6, the coverages of Books and Movie were higher than that of Electronics and Tools. Note that nearly 90 % of users in Books could get predictions by DA, UA and CA using 20 % $\sim$ 60% communities, whereas only 40 % or less users could get predictions by NN and LFM. In Books, many users have strong relationships with each other (i.e., a lot of interactions throughout various items) so that many commenters without previous ratings can get predictions by average based approaches, whereas users without previous ratings can not get predictions by NN.



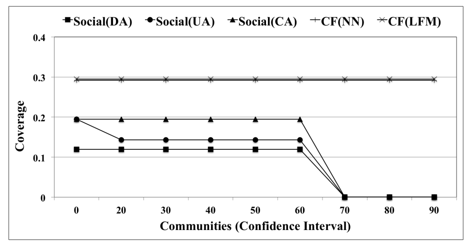
(a) Books coverage



(b) Movie coverage



(c) Electronics coverage



(d) Tools coverage

Fig. 6. Coverage in each category

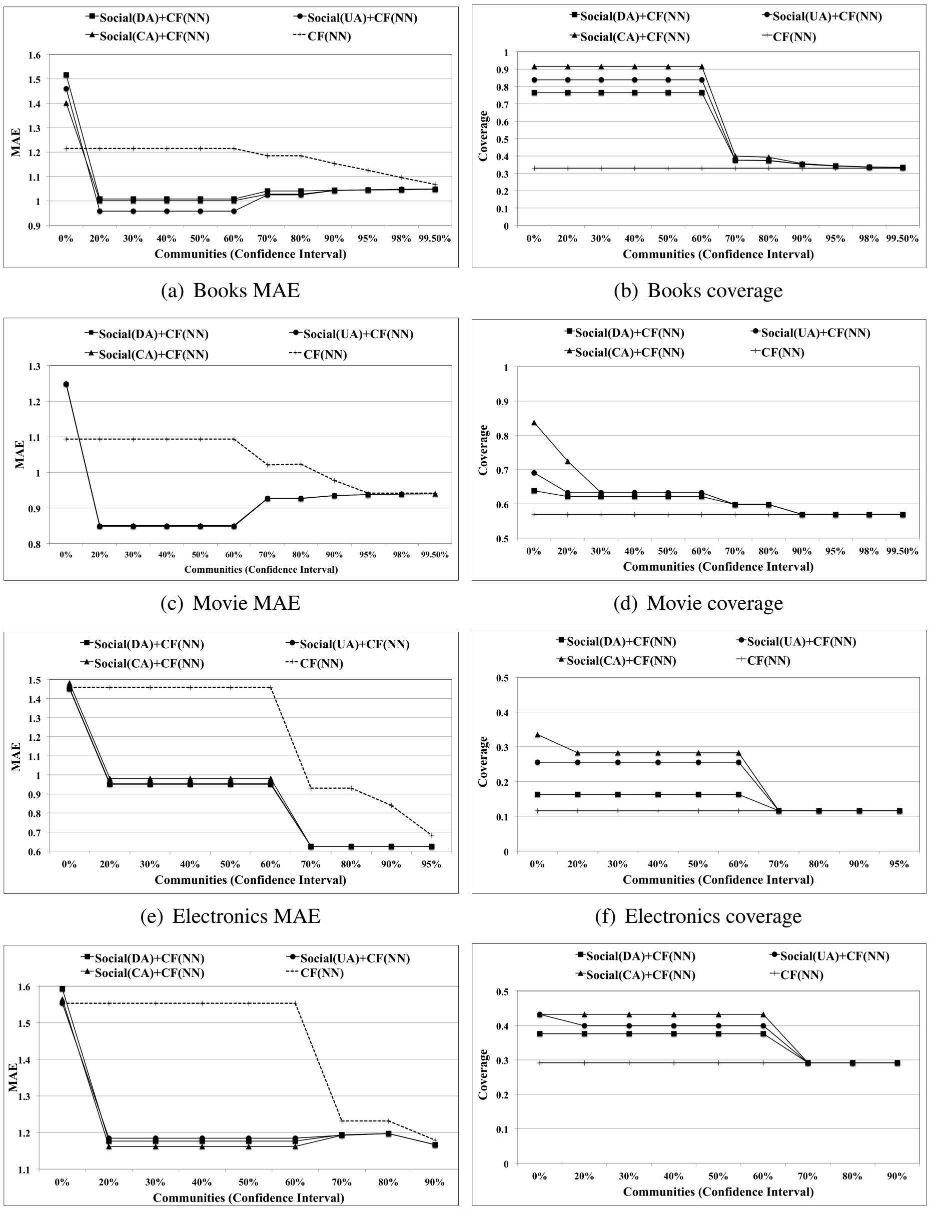
As discussed before, interactions in Electronics and Tools are similar to random interactions; which means users do not have many interactions throughout various Electronics and Tools items (i.e., lack of implicit relationships). Hence, many commenters without previous ratings could not benefit from communities much using average-based approaches (worse coverage).

Overall, Fig.6 shows that the coverage decreases as τ increases in general while as shown in Fig.5, prediction accuracy improves as $\tau\%$ increases. Although we get the highest accuracy using stronger relations, the coverage may be too small, close to 0%. To address this issue, we combined our social recommender system using discovered communities with existing collaborative filtering methods, which will be described in the following section.

6.3. Combining A Social Recommender System with Collaborative Filtering Methods

Note that our goal is not to propose a new recommender system, but to suggest incorporating discovered implicit communities into existing recommender systems. As mentioned above, there may not exist a large community and a lot of users may not get predictions in certain categories such as Electronics and Tools. To balance accuracy and coverage moderately, we combined our social recommender system using discovered communities with existing collaborative methods, called a *hybrid approach*. That is, if a user belongs to a community, a social recommendation algorithm is applied, otherwise a collaborative algorithm is applied. As shown in figures 5 and 6, there was no big difference between NN and LFM in terms of MAE and coverage. We thus only present results with one of collaborative filtering methods, NN, for our hybrid approach in this section.

Fig.7 shows MAE and coverage when we use hybrid approaches. Overall, our results showed that our hybrid approaches beat the collaborative method NN in both prediction accuracy and coverage across all four categories. As mentioned above, there are fewer users in a larger $\tau\%$ community so that many users got predictions by NN. Consequently, MAE of the hybrid approach converged to MAE of NN, as τ increases as shown in Fig.7. Note that communities in Tools were the smallest among 4 categories. Hence, MAE of the hybrid approach was similar to MAE of NN even for 70% or 80% communities; whereas MAE of the hybrid approach greatly improved over NN in Books, Movie, and Electronics.



(a) Books MAE (b) Books coverage

(c) Movie MAE (d) Movie coverage

(e) Electronics MAE (f) Electronics coverage

(g) Tools MAE (h) Tools coverage

Fig. 7. MAE and Coverage of a hybrid approach in each category

As shown in Fig.5 and Fig.6, an undirected approach covers more users than a directed approach and there is no significant difference between MAE of those. In other words, more users will get predictions by social recommendation algorithms; and fewer users will get predictions by NN, when an undirected approach is applied. Hence, MAE of the undirected approach with NN is lower than the directed approach with NN.

On the other hand, the coverage of the hybrid approach with DA, UA, and CA showed an increase over NN as shown in Fig.7. This is because commenters without previous ratings can get predictions by average-based approaches, whereas NN can only cover users with previous ratings.

As discussed before, each user tended to have many interactions as well as to write many reviews in Books and Movie. That is, many of covered users by social recommendation algorithms and NN were overlapped. Hence, the coverage of the hybrid approach was the same as or similar to the larger one of two (i.e., either coverage of the social recommendation or NN) as shown in figures 6 and 7.

On the other hand, many reviewers in Electronics and Tools did not belong to discovered communities. That is, most users covered by the social recommender algorithms were commenters without previous ratings. Accordingly, users covered by social recommender algorithms and users covered by NN were almost mutually exclusive. Hence, covered users by the hybrid approach were union of two nearly disjoint users. That is, the coverage of the hybrid approach was similar to the sum of two coverages (i.e., coverages of social recommender algorithms and NN) as shown in figures 6 and 7.

In short, Fig.7 shows that we could adjust the trade-off between the data sparsity problem and prediction accuracy with different τ. Indeed, it was shown that the hybrid approach using 20 % $\sim$ 60% communities greatly improved both accuracy and coverage over NN. In fact, the same results hold for LFM.

7. CONCLUSION

Recent research has pointed out the importance of social relationships in recommender systems based on an intuition that friends are likely to have similar interests to each other. However, social relationships are not always available in recommender systems and it is not guaranteed that friends always share similar interests in different categories of items.

In this paper, we proposed a method to extract implicit relationships from a system and to utilize them for recommendation. Based on the assumption that if users have beyond-random interactions with each other, they are likely to have similar interests, we exposed implicit communities in the system. In order to corroborate that our approach works in real recommender systems, we crawled Amazon and performed analysis on the dataset. With experimental results, we showed that there exist implicit communities in Amazon, and that we can improve the recommendation accuracy by utilizing discovered communities.

REFERENCES

G. Adomavicius and A. Tuzhilin. 2005. Toward the Next Generation of Recommender Systems: A Survey of the State-of-the-Art and Possible Extensions. *IEEE Transactions on Knowledge and Data Engineering* (2005).

O. Arazy, N. Kumar, and B. Shapira. 2009. Improving Social Recommender Systems. *IT Professional* 11, 4 (2009), 38 –44.

P. Bonhard and M.A. Sasse. 2006. ‘Knowing me, knowing you’ Using profiles and social networking to improve recommender systems. *BT Technology Journal* 24 (2006), 84–98. Issue 3.

Yevgeniy Dodis and Adam Smith. 2005. Entropic security and the encryption of high entropy messages. *Theory of Cryptography* (2005), 556–577.

Tom DuBois, Jennifer Golbeck, John Kleint, and Aravind Srinivasan. 2009. Improving recommendation accuracy by clustering social networks with trust. *Recommender Systems & the Social Web* (2009), 1–8.

K. Funakoshi and T. Ohguro. 2000. A content-based collaborative recommender system with detailed use of evaluations. In *Knowledge-Based Intelligent Engineering Systems and Allied Technologies, 2000. Proceedings. Fourth International Conference on*, Vol. 1. 253 –256.

Jianming He and WesleyW. Chu. 2010. A Social Network-Based Recommender System (SNRS). 12 (2010), 47–74.

Jonathan Herlocker, Joseph Konstan, Loren Terveen, and John Riedl. 2004. Evaluating collaborative filtering recommender systems. *ACM Trans. Inf. Syst.* 22, 1 (2004), 5–53.

Thomas Hofmann. 2004. Latent semantic models for collaborative filtering. *ACM Transactions on*

Information Systems (TOIS) 22, 1 (2004), 89–115.
Zan Huang, D. Zeng, and Hsinchun Chen. 2007. A Comparison of Collaborative-Filtering Recommendation Algorithms for E-commerce. *Intelligent Systems, IEEE* 22, 5 (2007), 68 –78.
Meng Jiang, Peng Cui, Rui Liu, Qiang Yang, Fei Wang, Wenwu Zhu, and Shiqiang Yang. 2012. Social contextual recommendation. In *Proceedings of the 21st ACM international conference on Information and knowledge management*. ACM, 45–54.
Joseph A. Konstan. 2004. Introduction to recommender systems: Algorithms and Evaluation. *ACM Trans. Inf. Syst.* 22, 1 (2004), 1–4.
Yehuda Koren. 2010. Factor in the neighbors: Scalable and accurate collaborative filtering. *ACM Transactions on Knowledge Discovery from Data (TKDD)* 4, 1 (2010), 1.
Yehuda Koren, Robert Bell, and Chris Volinsky. 2009. Matrix factorization techniques for recommender systems. *Computer* 42, 8 (2009), 30–37.
Yun-Jung Lee, Eun-Kyung Kim, Hwan-Gue Cho, and Gyun Woo. 2012. Detecting and visualizing online dispute dynamics in replying comments. *Software: Practice and Experience* (2012).
Damien Leprovost, Lylia Abrouk, and David Gross-Amblard. 2012. Discovering implicit communities in web forums through ontologies. *Web Intelligence and Agent Systems* 10, 1 (2012), 93–103.
G. Linden, B. Smith, and J. York. 2003. Amazon.com recommendations: item-to-item collaborative filtering. *Internet Computing, IEEE* 7, 1 (2003), 76 – 80.
Di Ma, Dandan Song, and Lejian Liao. 2013. Incorporating social actions into recommender systems. In *Web-Age Information Management*. Springer, 698–704.
Hao Ma, Dengyong Zhou, Chao Liu, Michael R Lyu, and Irwin King. 2011. Recommender systems with social regularization. In *Proceedings of the fourth ACM international conference on Web search and data mining*. ACM, 287–296.
Ashwin Machanavajjhala, Aleksandra Korolova, and Atish Das Sarma. 2011. Personalized social recommendations: accurate or private. *Proc. VLDB Endow.* 4, 7 (April 2011), 440–450.
Raymond J. Mooney and Loriene Roy. 2000. Content-based book recommending using learning for text categorization. In *Proceedings of the fifth ACM conference on Digital libraries*. ACM, 195–204.
M. E. J. Newman, S. H. Strogatz, and D. J. Watts. 2001. Random graphs with arbitrary degree distributions and their applications. *Phys. Rev. E* 64 (Jul 2001), 026118. Issue 2.
MichaelJ. Pazzani and Daniel Billsus. 2007. Content-Based Recommendation Systems. 4321 (2007), 325–341.
Paul Resnick and Hal R. Varian. 1997. Recommender systems. *Commun. ACM* 40, 3 (1997), 56–58.
Francesco Ricci, Lior Rokach, and Bracha Shapira. 2011. Introduction to Recommender Systems Handbook. (2011), 1–35.
Badrul Sarwar, George Karypis, Joseph Konstan, and John Riedl. 2001. Item-based collaborative filtering recommendation algorithms. In *Proceedings of the 10th international conference on World Wide Web (WWW '01)*. 285–295.
J.Ben Schafer, Dan Frankowski, Jon Herlocker, and Shilad Sen. 2007. Collaborative Filtering Recommender Systems. 4321 (2007), 291–324.
Anne Schuth, Maarten Marx, and Maarten de Rijke. 2007. Extracting the discussion structure in comments on news-articles. In *Proceedings of the 9th annual ACM international workshop on Web information and data management*. ACM, 97–104.
Erez Shmueli, Amit Kagian, Yehuda Koren, and Ronny Lempel. 2012. Care to comment?: recommendations for commenting on news stories. In *Proceedings of the 21st international conference on World Wide Web*. ACM, 429–438.
Richard C Sprinthall and Stephen T Fisk. 1990. *Basic statistical analysis*. Prentice Hall Englewood Cliffs, NJ.
Bin Xu, Jiajun Bu, Chun Chen, and Deng Cai. 2012. An exploration of improving collaborative recommender systems via user-item subgroups. In *Proceedings of the 21st international conference on World Wide Web*. ACM, 21–30.